| Centre No. | | | | | | Paper Reference | Surname | Initial(s) |
|---|---|---|---|---|---|---|---|---|
| Candidate No. | | | | | | 6 6 6 9 / 0 1 | Signature | |

Paper Reference(s)

# 6669/01

# Edexcel GCE

## Further Pure Mathematics FP3

## Advanced/Advanced Subsidiary

Monday 23 June 2014 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

| Question Number | Leave Blank |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| Total | |

| **Materials required for examination** | **Items included with question papers** |
|---|---|
| Mathematical Formulae (Pink) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.**

### Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.



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PEARSON

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1. The line $l$ passes through the point $P(2, 1, 3)$ and is perpendicular to the plane $\Pi$ whose vector equation is

$$\mathbf{r}.(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) = 3$$

Find

(a) a vector equation of the line $l$,

**(2)**

(b) the position vector of the point where $l$ meets $\Pi$.

**(4)**

(c) Hence find the perpendicular distance of $P$ from $\Pi$.

**(2)**

Leave blank

**Question 1 continued**

**(Total 8 marks)**

Q1

Leave blank

**2.**

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 4 & 1 \\ 0 & 5 & 0 \end{pmatrix}$$

(a) Show that matrix **M** is not orthogonal.

**(2)**

(b) Using algebra, show that 1 is an eigenvalue of **M** and find the other two eigenvalues of **M**.

**(5)**

(c) Find an eigenvector of **M** which corresponds to the eigenvalue 1

**(2)**

The transformation $M : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is represented by the matrix **M**.

(d) Find a cartesian equation of the image, under this transformation, of the line

$$x = \frac{y}{2} = \frac{z}{-1}$$

**(4)**

Leave blank

**Question 2 continued**



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**Question 2 continued**



P 4 3 1 4 7 A 0 6 3 2

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**Question 2 continued**

Q2

**(Total 13 marks)**





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**3.** Using calculus, find the exact value of

(a) $\displaystyle\int_1^2 \frac{1}{\sqrt{(x^2 - 2x + 3)}}\,\mathrm{d}x$ **(4)**

(b) $\displaystyle\int_0^1 \mathrm{e}^{2x} \sinh x \,\mathrm{d}x$ **(4)**

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**Question 3 continued**

Leave blank

**Question 3 continued**





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**Question 3 continued**

**(Total 8 marks)**

Q3

Leave blank

**4.** Using the definitions of hyperbolic functions in terms of exponentials,

(a) show that

$$\text{sech}^2 x = 1 - \tanh^2 x$$

**(3)**

(b) solve the equation

$$4\sinh x - 3\cosh x = 3$$

**(4)**





P 4 3 1 4 7 A 0 1 2 3 2

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**Question 4 continued**

Q4

**(Total 7 marks)**



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**Turn over**

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**5.** Given that $y = \text{artanh}\dfrac{x}{\sqrt{(1+x^2)}}$

show that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{(1+x^2)}}$

**(4)**

Leave blank

**Question 5 continued**

Q5

**(Total 4 marks)**







P 4 3 1 4 7 A 0 1 5 3 2

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**6.** [*In this question you may use the appropriate trigonometric identities on page 6 of the pink Mathematical Formulae and Statistical Tables*.]

The points $P(3\cos\alpha, 2\sin\alpha)$ and $Q(3\cos\beta, 2\sin\beta)$, where $\alpha \neq \beta$, lie on the ellipse with equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

(a) Show the equation of the chord $PQ$ is

$$\frac{x}{3}\cos\frac{(\alpha+\beta)}{2} + \frac{y}{2}\sin\frac{(\alpha+\beta)}{2} = \cos\frac{(\alpha-\beta)}{2}$$

**(4)**

(b) Write down the coordinates of the mid-point of $PQ$.

**(1)**

Given that the gradient, $m$, of the chord $PQ$ is a constant,

(c) show that the centre of the chord lies on a line

$$y = -kx$$

expressing $k$ in terms of $m$.

**(5)**





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**Question 6 continued**



P 4 3 1 4 7 A 0 1 7 3 2

**Turn over**

Leave blank

**Question 6 continued**

Leave blank

**Question 6 continued**

Q6

**(Total 10 marks)**

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**7.** A circle $C$ with centre $O$ and radius $r$ has cartesian equation $x^2 + y^2 = r^2$ where $r$ is a constant.

(a) Show that $1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \frac{r^2}{r^2 - x^2}$

**(3)**

(b) Show that the surface area of the sphere generated by rotating $C$ through $\pi$ radians about the $x$-axis is $4\pi r^2$.

**(5)**

(c) Write down the length of the arc of the curve $y = \sqrt{(1 - x^2)}$ from $x = 0$ to $x = 1$

**(1)**

Leave blank

**Question 7 continued**

Leave blank

**Question 7 continued**

**Question 7 continued**

Leave blank

(Total 9 marks)

Q7



**Turn over**

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**8.** The position vectors of the points $A$, $B$ and $C$ from a fixed origin $O$ are

$$\mathbf{a} = \mathbf{i} - \mathbf{j}, \ \mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}, \ \mathbf{c} = 2\mathbf{j} + \mathbf{k}$$

respectively.

(a) Using vector products, find the area of the triangle $ABC$.

**(4)**

(b) Show that $\frac{1}{6}\mathbf{a}.(\mathbf{b} \times \mathbf{c}) = 0$

**(3)**

(c) Hence or otherwise, state what can be deduced about the vectors **a**, **b** and **c**.

**(1)**

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**Question 8 continued**



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**Question 8 continued**





P 4 3 1 4 7 A 0 2 6 3 2

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**Question 8 continued**

Q8

**(Total 8 marks)**



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**9.** $$I_n = \int (x^2 + 1)^{-n}\, \mathrm{d}x, \quad n > 0$$

(a) Show that, for $n > 0$

$$I_{n+1} = \frac{x(x^2 + 1)^{-n}}{2n} + \frac{2n - 1}{2n} I_n$$

**(5)**

(b) Find $I_2$

**(3)**

Leave blank

**Question 9 continued**



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**Turn over**

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**Question 9 continued**

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**Question 9 continued**



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**Turn over**

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**Question 9 continued**

Q9

**(Total 8 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**